1. Suppose we observe $Y_1, \ldots, Y_N$ i.i.d. with $Y_i \mid \lambda, \beta \sim \text{Pois}(\lambda)$ and where $\lambda \mid \beta \sim \text{gamma}(2, \beta)$ and $\beta \sim \text{gamma}(a, b)$ where $a, b$ are two fixed constants.

a) Find the full conditional distributions $\lambda \mid \beta, \vec{Y}$ and $\beta \mid \lambda, \vec{Y}$ where $\vec{Y} = (Y_1, \ldots, Y_N)$. Note that the calculation will be VERY similar to what you did for last weeks homework.

b) Suppose that $N = 4$ and you observe $\vec{Y} = (8, 9, 9, 10)$. Write an R program to find the joint posterior distribution $\lambda, \beta \mid \vec{Y}$ using Gibbs sampling. Do this twice: first using $a = b = 1$ and again using $a = b = .001$.

In each case, display histograms of the posterior distributions for $\lambda$ and $\beta$ and calculate 95% credible intervals. Discuss any similarities and/or differences you find using the different values for $(a, b)$.

Careful: Note that the default parametrization in R for the gamma density uses the scale parametrization, whereas we use the rate parametrization. You’ll need to use the ‘rate=’ option in the 
\texttt{rgamma} function.