This week's homework is going to be very similar to last week's homework, except this time you are going to write BUGS code to do the analysis.

1. Suppose we observe \( Y_1, \ldots, Y_N \) i.i.d. with \( Y_i \mid \lambda, \beta \sim \text{Pois}(\lambda) \) and where \( \lambda \mid \beta \sim \text{Gamma}(2, \beta) \) and \( \beta \sim \text{Gamma}(a, b) \) where \( a, b \) are two fixed constants.

   a) Find the full conditional distributions \( \lambda \mid \beta, \bar{Y} \) and \( \beta \mid \lambda, \bar{Y} \) where \( \bar{Y} = (Y_1, \ldots, Y_N) \). Note that this question was asked on last week's homework. Those of you that did it correctly can repeat what you wrote last week. For the others, please try again.

   b) Suppose that \( N = 4 \) and you observe \( \bar{Y} = (8, 9, 9, 10) \). Write an R program and BUGS code to simulate the joint posterior distribution \( \lambda, \beta \mid \bar{Y} \). Do this twice: first using \( a = b = 1 \) and again using \( a = b = .001 \). Note that you don’t need your result from part a) to do this, as WinBUGS will figure out the full conditionals. This is in contrast to last week where you wrote your own Gibbs sampler and needed to know the full conditionals.

In each case, display histograms of the posterior distributions for \( \lambda \) and \( \beta \) and calculate 95% credible intervals. Discuss any similarities and/or differences you find using the different values for \( (a, b) \). HINT: How does the mean and variance of the prior for \( \beta \) differ when \( a = b = 1 \) versus \( a = b = .001 \)?